Optimal Reads-From Consistency Checking for C11-Style Memory Models

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Overview

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$$x := 0, y := 0$$

Thread 1	**Thread 2**
$x := 1$	$y := 1$
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Scenario II

$$x := 0, y := 0$$

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$x := 1$	$y := 1$
$a := y$	$b := x$

Under sequential consistency:

- Is *Scenario I* possible?

  $\hookrightarrow a = b = 0$ ✗

- Is *Scenario II* possible?

  $\hookrightarrow a = b = 1$

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- Well understood for traditional memory models
  $\hookrightarrow$ Sequential consistency, x86-TSO
- Little is known about the variants of C11 memory model
  $\hookrightarrow$ **This work fills this gap!**

Contributions

- We study reads-from consistency checking in various variants of C11
- **Main results:**
  - Efficient algorithms

    $\hookrightarrow$ Optimal or nearly-optimal
  - Complexity characterization

    $\hookrightarrow$ Fine-grained optimality or $\mathcal{NP}$-hardness results
  - Empirical evaluation

    $\hookrightarrow$ Shows the impact of new algorithms in practice

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- Support for low-level atomic operations
  - ↪ Load, Store, RMW
  - ↪ Used for communication between threads
- Memory accesses levels:
  - ↪ Synchronization guarantees
  - ↪ Implementation cost

```
atomic<int> x (0);
x.store(1, memory_order_relaxed);
x.load(memory_order_acquire);
```

Sequentially consistent

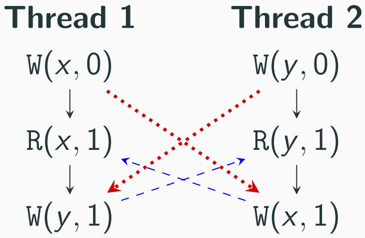
Release Acquire

Relaxed

C11 Memory Model

- Semantics of a program is defined as a set of consistent executions
- Each execution is a graph
  - ↪ Nodes are instructions in the program
  - ↪ Edges represent certain relations among the instructions

Thread 1	**Thread 2**
$x := 0;$	$y := 0;$
$a := x;$	$b := y;$
$y := 1;$	$x := 1;$



C11 Memory Model

- Standard relations:
  $\hookrightarrow$ Program order (po): Precedence among the same thread events

Thread 1	**Thread 2**
$x := 0;$	$y := 0;$
$a := x;$	$b := y;$
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$$\begin{array}{cc}
\textbf{Thread 1} & \textbf{Thread 2} \\
\mathtt{W}(x,0) & \mathtt{W}(y,0) \\
\text{po}\downarrow & \text{po}\downarrow \\
\mathtt{R}(x,?) & \mathtt{R}(y,?) \\
\text{po}\downarrow & \text{po}\downarrow \\
\mathtt{W}(y,1) & \mathtt{W}(x,1)
\end{array}$$

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  - ↪ Program order (po): Precedence among the same thread events
  - ↪ Reads-from (rf): Relates the writes to the loads which read their value

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\begin{tikzcd}
\mathtt{W}(x,0) \arrow[d] & & \mathtt{W}(y,0) \arrow[d] \\
\mathtt{R}(x,1) \arrow[d] & & \mathtt{R}(y,1) \arrow[d] \\
\mathtt{W}(y,1) \arrow[urr, dashed, "rf"'] & & \mathtt{W}(x,1) \arrow[ull, dashed, "rf"]
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C11 Memory Model

- Standard relations:
  - $\hookrightarrow$ Program order (po): Precedence among the same thread events
  - $\hookrightarrow$ Reads-from (rf): Relates the writes to the loads which read their value
  - $\hookrightarrow$ Modification order (mo): A total order of the writes on a given location

Thread 1	**Thread 2**
$x := 0;$	$y := 0;$
$a := x;$	$b := y;$
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$$
\begin{tikzcd}
\textbf{Thread 1} & \textbf{Thread 2} \\
\mathtt{W}(x, 0) \arrow[d] \arrow[ddr, dotted, "mo"] & \mathtt{W}(y, 0) \arrow[d] \arrow[ddl, dotted, "mo"'] \\
\mathtt{R}(x, ?) \arrow[d] & \mathtt{R}(y, ?) \arrow[d] \\
\mathtt{W}(y, 1) & \mathtt{W}(x, 1)
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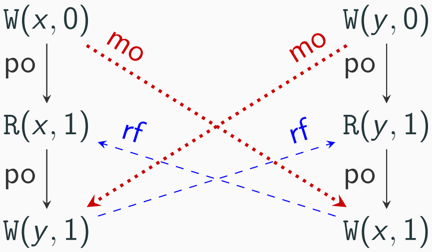
C11-Style Memory Models

- A memory model restricts which executions are consistent



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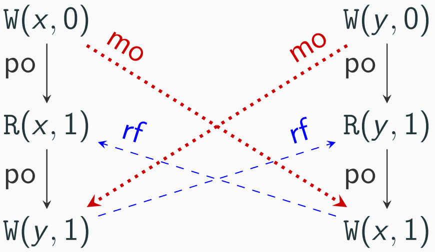

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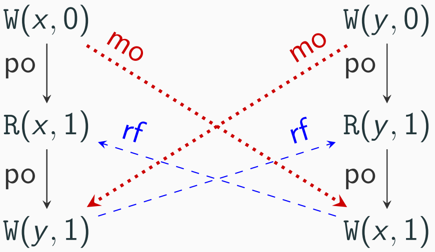

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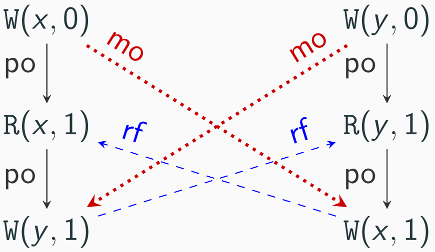

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Memory model $\mathcal{M}1$: $\mathrm{acyclic}(\mathrm{po} \cup \mathrm{mo})$

Memory model $\mathcal{M}2$: $\mathrm{acyclic}(\mathrm{po} \cup \mathrm{mo}) \wedge \mathrm{acyclic}(\mathrm{po} \cup \mathrm{rf})$

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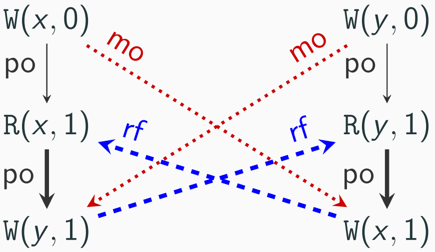

$\rightarrow$ **Consistent in $\mathcal{M}1$ ✓**

$\rightarrow$ **Not consistent in $\mathcal{M}2$ ✗**

Memory model $\mathcal{M}1$: $\text{acyclic}(\text{po} \cup \text{mo})$

Memory model $\mathcal{M}2$: $\text{acyclic}(\text{po} \cup \text{mo}) \wedge \text{acyclic}(\text{po} \cup \text{rf})$

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- **Input:** A partial execution X in a memory model $\mathcal{M}$
  - $\hookrightarrow$ X contains po and rf
  - $\hookrightarrow$ X lacks mo
- **Task:** Check if X can be extended to a complete execution consistent in $\mathcal{M}$
  - $\hookrightarrow$ Find an mo that turns X consistent

RC20 Memory Model

- Captures a rich fragment of C11
  - ↪ Contains `Release`, `Acquire` and `Relaxed` accesses
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- **Previous works:** $O(n^3 \cdot k)$, $O(n^2 \cdot k)$
  - $\hookrightarrow$ For n events and k threads

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- **Previous works:** $O(n^3 \cdot k)$, $O(n^2 \cdot k)$
  - $\hookrightarrow$ For n events and k threads
- **Our result:** $O(n \cdot k)$
  - $\hookrightarrow$ Key idea: *minimal coherence* witness relation

Witness Relation

Witness relation

- ↪ Serves as a witness for consistency
- ↪ Construct a partially ordered mo
    - ↪ Include *necessary* orderings enforced by the memory model
    - ↪ It should be extendable to a total mo

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$\hookrightarrow$ Construct a partially ordered mo

- $\hookrightarrow$ Include *necessary* orderings enforced by the memory model
- $\hookrightarrow$ It should be extendable to a total mo

Recall $\rightarrow$

Reads-from consistency checking:

$\hookrightarrow$ Task is to find an mo

$\hookrightarrow$ mo is a total order on the same location writes

Witness Relation

$$
\begin{tikzcd}
\mathtt{W}_1(x) \arrow[dr, dashed, "rf"] & \mathtt{W}_2(x) \arrow[d, "\mathrm{po}"] & \mathtt{W}_3(x) \\
& \mathtt{R}(x) &
\end{tikzcd}
$$

Witness Relation

$$
\begin{tikzcd}
\mathtt{W}_1(x) \arrow[dr, dashed, "rf"] & \mathtt{W}_2(x) \arrow[l, dotted, "mo"'] \arrow[d, "po"] & \mathtt{W}_3(x) \\
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- $\mathtt{W}_2(x)$ must be mo ordered before $\mathtt{W}_1(x)$

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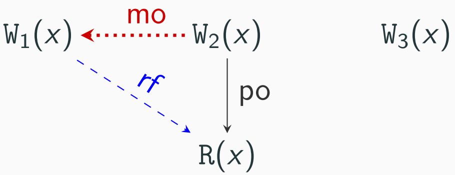

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- $\mathtt{W}_2(x)$ must be mo ordered before $\mathtt{W}_1(x)$
- $\mathtt{W}_3(x)$ is not relevant
  - $\hookrightarrow$ It can be left unordered
- Witness should always be extendable to a total mo
  - $\hookrightarrow$ $\mathtt{W}_3(x) \xrightarrow{\mathsf{mo}} \mathtt{W}_2(x)$
  - $\hookrightarrow$ $\mathtt{W}_1(x) \xrightarrow{\mathsf{mo}} \mathtt{W}_3(x)$

Minimal Coherence for RC20

Minimal coherence

$\hookrightarrow$ Serves as a witness for consistency

$\hookrightarrow$ Weaker than prior witness relations

$\hookrightarrow$ Allows efficient consistency checking algorithm for RC20

Minimal Coherence for RC20

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$$
\begin{tikzcd}
\mathtt{W}(x) \arrow[d, dashed, "\mathsf{rf}"'] & & \\
\mathtt{RMW}(x) \arrow[r, dashed, "\mathsf{rf}"] \arrow[d, dashed, "\mathsf{rf}"'] & \mathtt{R}(x) \arrow[d, "\mathsf{po}"] & \mathtt{W}(x) \arrow[dl, dashed, "\mathsf{rf}"'] \\
\mathtt{RMW}(x) & \mathtt{RMW}(x) &
\end{tikzcd}
$$

Prior witness relations

$$
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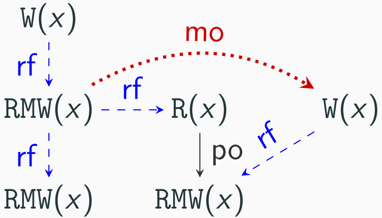

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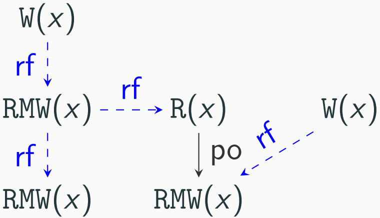

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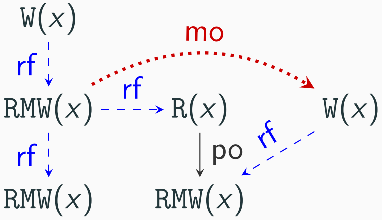

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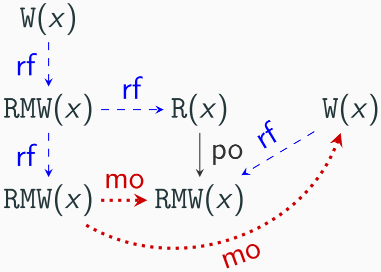

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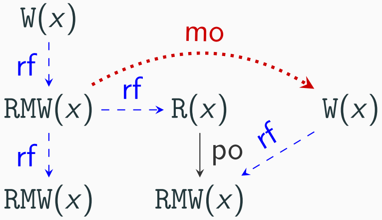

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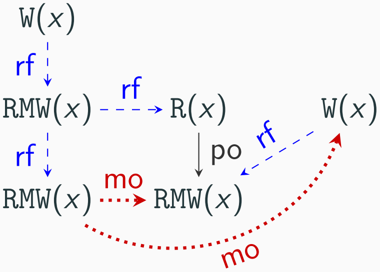

- Minimal coherence is weaker!

  ↪ More efficient to compute

Experimental Results

- Focused on the RC20/Release-Acquire (RA) fragments
  - $\hookrightarrow$ RA is a fragment of RC20
- Performed an evaluation in two scenarios
  - $\hookrightarrow$ Model checking
  - $\hookrightarrow$ Testing
- Modified only the consistency checking components

Experimental Results - Model Checking

- Implemented minimal coherence inside GenMC[1]
- Compared with the original GenMC
  - ↪ 25 standard benchmarks
  - ↪ 2 hour timeout

	GenMC	**Our Algorithm**
Average time per execution	14.5 sec	0.26 sec
Total number of executions	356K	4.6M

[1] Michalis Kokologiannakis, Viktor Vafeiadis. GenMC: A Model Checker for Weak Memory Models. CAV'21

Experimental Results - Testing

- Implemented minimal coherence inside C11Tester[1]
  - ↪ Online version
  - ↪ $O(n \cdot k)$ bound does not apply
- Compared with the original C11Tester
  - ↪ 32 standard benchmarks

	C11Tester	**Our Algorithm**
Total analysis time	286 sec	170 sec

[1]Weiyu Luo, Brian Demsky. C11Tester: a race detector for C/C++ atomics. ASPLOS'21

Summary of Established Bounds

- **NP-hard**: $O(k \cdot n^{k+1})$
  $\hookrightarrow$ Strong Release-Acquire (SRA)

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  $\hookrightarrow$ RC20
  $\hookrightarrow$ Weak Release-Acquire (WRA)
  $\hookrightarrow$ RMW-free SRA
- **Linear**: $O(n)$
  $\hookrightarrow$ Relaxed
- **Super-linear lower bound:**
  $\hookrightarrow$ RMW-free RA, SRA, WRA
  $\hookrightarrow$ Improving $O(n \cdot k)$ bounds would be non-trivial

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Thank you!